### EHzürich

ANKOLE MELLON UNILERSITY

CRGH PENNSYLVANIA

### Slim NoC: A Low-Diameter On-Chip Network Topology for High Energy Efficiency and Scalability MACIEJ BESTA, SYED

MACIEJ BESTA, SYED MINHAJ HASSAN, SUDHAKAR YALAMANCHILI, RACHATA AUSAVARUNGNIRUN, ONUR MUTLU, TORSTEN HOEFLER

Georgia





Mary Market and Andrews and Andrews

## **MASSIVELY PARALLEL MANYCORES**



### SW26010: 260 cores

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#### PEZY-SC2: 2048 cores



### SW26010: 260 cores





#### PEZY-SC2: 2048 cores



### SW26010: 260 cores



### Adapteva Epiphany: 1024 cores





#### PEZY-SC2: 2048 cores



### SW26010: 260 cores

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### Adapteva Epiphany: 1024 cores













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## NETWORKS IN COMPUTE CLUSTERS





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DRAGONFLY, SLIM FLY



[1] C. E. Leiserson. Fat-trees: Universal Networks for Hardware-Efficient Supercomputing. IEEE Transactions on Computers. 1985.

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DRAGONFLY, SLIM FLY **Dragonfly** [2] Fat tree [1] diameter = 4 **TSUBAME2.0** diameter = 3 CASCADE **Cray Cascade** 

[1] C. E. Leiserson. Fat-trees: Universal Networks for Hardware-Efficient Supercomputing. IEEE Transactions on Computers. 1985.
 [2] J. Kim et al. Technology-Driven, Highly-Scalable Dragonfly Topology. ISCA'08.

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DRAGONFLY, SLIM FLY

### Fat tree [1]



diameter = 4

## Slim Fly [3] based on the Hoffman-Singleton Graph [4]: > ~50% fewer routers than Fat tree > ~30% fewer cables than Fat tree diar





diameter = 3

## diameter = 2

**TSUBAME2.0** 

Cray Cascade

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 A. J. Hoffman and R. R. Singleton. Moore graphs with diameter 2 and 3, IBM Journal of Research and Development. 1960.

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### **INSPIRATION: DIAMETER-2 SLIM FLY**

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### **INSPIRATION: DIAMETER-2 SLIM FLY**





## **Key** idea:



# Key idea:









# Key idea:





# Key idea:





# Key idea:





# Key idea:





# Key idea:





# Key idea:





# Key idea:





# Key idea:

















**Lower diameter and thus average path length**: fewer needed links / routers.



### 🛠 Key method

**Optimize towards the Moore Bound [1]**: the upper bound on the *number of vertices* in a graph with given *diameter D* and *radix k*.

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**Optimize towards the Moore Bound [1]**: the upper bound on the *number of vertices* in a graph with given *diameter D* and *radix k*.

MooreBound(D,k) = 1 + k + k(k-1)

+ k(k - 1)<sup>2</sup> + ...
Thus, Slim Fly ensures
the lowest radix (port count)
for a given node count
and for a fixed diameter...
Sounds ideal for an on-chip setting?



Y



WHY NOT JUST USE SLIM FLY AS AN ON-CHIP NETWORK?





and and the

### SLIM FLY ON CHIP – FIRST ATTEMPT STRUCTURE INTUITION

Example design for *diameter* = 2



Example design for *diameter* = 2



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Example design for *diameter* = 2







Example design for *diameter* = 2







Example design for *diameter* = 2

A subgraph with identical groups of routers







Example design for *diameter* = 2

A subgraph with identical groups of routers







Example design for *diameter* = 2





P. Landa





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Groups form a fully-connected bipartite graph





[1] J. Kim, W. J. Dally, D. Abts. Flattened butterfly: a cost-efficient topology for high-radix networks. ISCA'07



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Bad! No clear advantages from a topology that is close-to-optimal in the radix-size-diameter tradeoff

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# What are the problems with the simple on-chip Slim Fly?

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Near-best radix-size-diameter tradeoff, but...



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Near-best radix-size-diameter tradeoff, but...



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Near-best radix-size-diameter tradeoff, but...



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Near-best radix-size-diameter tradeoff, but...



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Short wire: small

input buffers

Near-best radix-size-diameter tradeoff, but...

Long wire: traversing the whole die requires large input buffers for full link utilization



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#### PROBLEMS WITH SIMPLE ON-CHIP SLIM FLY, PART 2



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#### PROBLEMS WITH SIMPLE ON-CHIP SLIM FLY, PART 2

Network radix k'	Concen- tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right ceil^{**}$	Network size N	Router count N <sub>r</sub>	Input param. <i>q</i>
3	2	100%	16	8	2
5	2	66%	36	18	3
5	3	100%	54	18	3
5	4	133%	72	18	3
7	3	75%	150	50	5
7	4	100%	200	50	5
7	5	120%	250	50	5
11	4	66%	392	98	7
11	5	83%	490	98	7
11	6	100%	588	98	7
11	7	116%	686	98	7
11	8	133%	784	98	7

Paker Participation and the

#### PROBLEMS WITH SIMPLE ON-CHIP SLIM FLY, PART 2

Various Slim Fly configurations

Network radix k'	Concen- tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right ceil **$	Network size N	Router count N <sub>r</sub>	Input param. q
3	2	100%	16	8	2
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#### PROBLEMS WITH SIMPLE ON-CHIP SLIM FLY, PART 2

Various Slim Fly configurations

Network radix k'	Concen- tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right ceil **$	Network size N	Router count N <sub>r</sub>	Input param. q
		No noo	dto po	50attant	ion
		no nee	u to pa	yalleni	lion
		<sup>8</sup> to all t	hoso n	mbors	$\bigcirc$
			liese ii	unners	



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# PROBLEMS WITH SIMPLE ON-CHIP SLIM FLY, PART 2

Various Slim Fly configurations			? Are there configurations with				
Network radix k'	Concen- tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right ceil^{**}$	Network size N	Router count N <sub>r</sub>	Input param. q		
		No noo	d to na	$\sqrt{2}^{0}$ attont	·ion		
		NO HEE	u lu pa	yallem	JUII		
		to all t	hose n	imhers	$\odot$		
		10 <b>0%</b> and		unibers			



PROBLEMS WITH SIMPLE ON-CHIP SLIM FLY, PART 2								
	Various Sli configurat	m Fly tions	? conf	with	be			
Network radix k'	Concen- tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right ceil^{**}$	Network size N	Router count N <sub>r</sub>	Input param.	t . <b>q</b>		
3	2	100%	16	8	2			
		No noo	25to no	50 attant	ion			
		no nee	a lo pa	yattent	,ION			
			those n	mborc	$\bigcirc$			
			liese n	umpers				

...number of nodes/routers being a power of two?

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PROBLEMS WITH SIMPLE ON-CHIP SLIM FLY, PART 2r							nber of nodes/routers
	Various Sli	m Fly	?	Are there		bei	ng a power of two?
	configurat	ions	conf	igurations	with		equally many cores on each die side?
Network radix <i>k'</i>	Concen- tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right ceil^{**}$	Network size N	Router count N <sub>r</sub>	Input param.	q	
3	2	100%	16	8	2		
		No nee	d to pa	y attent	ion		
		to all t	these n	umbers	$\odot$		
		100%					



PROBLEMS WITH SIMPLE ON-CHIP SLIM FLY, PART 2n							nber of nodes/routers
	Various Slim Fly		?	Are there		bei	ng a power of two?
	configurat	tions	conf	igurations	with		equally many cores on each die side?
Network radix k'	Concen- tration p	$p/\left\lceil \frac{k'}{2} \right\rceil^{**}$	Network size N	Router count N <sub>r</sub>	Inpu param	t . q	equally many routers
							on cach are side.
		No nee	d to pa	y attent	tion		
		to all t	these n	umbers	$\odot$		

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PROBLEMS WITH SIMPLE ON-CHIP SLIM FLY, PART 2							number of nodes/routers	
	Various Slim Fly		?	Are there		being a power of two?		
	configurat	ions	confi	igurations v	with		equally many cores on each die side?	
radix k'	tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right\rceil^{**}$	Network size N	count $N_r$	param	t . <u>q</u>	equally many routers	
					2	_	on cach aic siac:	
					3 3 5	equal o	ly many router groups n each die side?	
		No nee	d to pa	y attent	ion			
		to all t	these n	umbers	$\odot$			

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PROBLEM	PROBLEMS WITH SIMPLE ON-CHIP SLIM FLY, PART 2number of nodes/routers							
	Various Sli	m Fly	?	Are there		b	eing a power of two?	
	configurat	ions	conf	igurations <b>v</b>	with		equally many cores on each die side?	
Network radix <i>k</i> '	Concen- tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right ceil^{**}$	Network size N	Router count N <sub>r</sub>	Inpu param	t . q	equally many routers	
			16				on each die side?	
			36 54 72		3 3 3 5	equa	ally many router groups on each die side?	
		No nee	d to pa	y attent	ion			
		to all t	these n	umbers	$\odot$			

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PROBLEM	is with <mark>S</mark> i	MPLE ON-	2	number of nodes/routers			
	Various Sli	m Fly	?	Are there		be	ing a power of two?
	configurat	ions	conf	igurations v	with		equally many cores on each die side?
Network radix k'	Concen- tration p	$p/\left\lceil \frac{k'}{2} \right\rceil^{**}$	Network size N	count N <sub>r</sub>	Input param	t . <u>q</u>	.equally many routers on each die side?
			36		3	_	
					3 •	equal	lly many router groups
					3	C	n each die side?
					5		
		No noo	d to pa	vottont		S The	re are few Slim Flies
		NO HEE	u lo pa	yalleni		that a	satisfy various on chin
		to all t	these n	umbers	$\odot$	tech	nological constraints
					7		

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## How to solve these problems?

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Short wire: small

input buffers

## SOLUTION: SLIM NOC Part I **NEW COST AND AREA MODELS, NEW LAYOUTS** Near-best radix-size-diameter tradeoff, but...



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input buffers

## SOLUTION: SLIM NOC Part I **NEW COST AND AREA MODELS, NEW LAYOUTS** Near-best radix-size-diameter tradeoff, but...



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Short wire: small

input buffers

## SOLUTION: SLIM NOC Part I New cost and area models, New layouts

radix-size-diameter tradeoff, but...



### SOLUTION: SLIM NOC Part I New cost and area models,

Short wire: small

input buffers

NEW LAYOUTS

Near-best radix-size-diameter tradeoff, but...











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## SOLUTION: SLIM NOC Part I

NEW COST AND AREA MODELS,

**NEW LAYOUTS** 





NEW COST AND AREA MODELS, NEW LAYOUTS



#### Minimize the average wire length (*M*) :

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$$M = \frac{\text{Sum of distances}}{\text{Number of links}}$$



NEW COST AND AREA MODELS, NEW LAYOUTS



#### Minimize the average wire length (*M*) :

$$M = \frac{\text{Sum of distances}}{\text{Number of links}}$$

#### Minimize the total buffer area ( $\Delta$ ):

$$\Delta = \sum_{\substack{\text{All router} \\ \text{pairs } i, j}}$$

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If *i*, *j* are connected, ( $\varepsilon_{ij} = 1$ ) add the size of a buffer from *i* to *j* 



NEW COST AND AREA MODELS, NEW LAYOUTS



ILP formulation: many more details for reproducibility and genericness, check the paper ⓒ



NEW COST AND AREA MODELS, NEW LAYOUTS



ILP formulation: many more details for reproducibility and genericness, check the paper  $\Phi(i, j) = 1$  if  $|x_i - x_j| > |y_i - y_j|$ , and 0 otherwise  $\Psi(i, j) = 1$  if  $|x_i - x_j| \le |y_i - y_j|$ , and 0 otherwise.

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 $\phi_{ij}(k, l) = \begin{cases} 1, \text{ if } k = x_i \land \min\{y_i, y_j\} \le l \le \max\{y_i, y_j\} \\ 1, \text{ if } l = y_j \land \min\{x_i, x_j\} \le k \le \max\{x_i, x_j\} \\ 0, \text{ otherwise} \end{cases}$ 

 $\psi_{ij}(k, l) = \begin{cases} 1, \text{ if } k = x_j \land \min\{y_i, y_j\} \le l \le \max\{y_i, y_j\} \\ 1, \text{ if } l = y_i \land \min\{x_i, x_j\} \le k \le \max\{x_i, x_j\} \\ 0, \text{ otherwise.} \end{cases}$ 

$$\sum_{i=1}^{N_r} \sum_{j=1}^{N_r} \varepsilon_{ij} [\phi_{ij}(k, l) \Phi(i, j) + \psi_{ij}(k, l) \Psi(i, j)] \le W$$



#### NEW COST AND AREA MODELS,

**NEW LAYOUTS** 





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**NEW LAYOUTS** 





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**NEW LAYOUTS** 





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NEW COST AND AREA MODELS, NEW LAYOUTS

Let us see some layouts

What difference do they make for lengths of wires?



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NEW COST AND AREA MODELS, NEW LAYOUTS





What difference do they make for lengths of wires? The "subgroup layout" (sn\_subgr) is best for 200 nodes The "group layout" (sn\_gr) is best for 1296 nodes (it reduces wiring complexity)



## Now let's move to the second problem...



SOLUTION: SLIM NOC Part II NON-PRIME FINITE FIELDS Are there configurations with			/ith	num beii	nber of nodes/routers ng a power of two?		
Various Slin	n Fly configu	rations				_	equally many cores on each die side?
Network C radix k' ti	Concen- ration p <sup>p</sup> /	$\left\lceil \frac{k'}{2} \right\rceil **$	Network size N	Router count N <sub>r</sub>	Input param. 2	$\frac{1}{2}$	equally many routers on each die side?
					3 3 3	equall. or	y many router groups n each die side?
	75% 100 120 66% 83% 100 116	No nee	d to pa	ay atten	tion	Ther that sa techr	re are few Slim Flies atisfy various on-chip hological constraints

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SOLUTIO Non-Prin	N: SLIM N ME FINITE FI	oc Part	II ? conf	Are there	with	number of nodes/routers being a power of two?
Various S	Slim Fly cor	nfigurations	Notwork	, Poutor	Innu	equally many cores on each die side?
radix k'	tration p	$\frac{p}{\left \frac{k'}{2}\right ^{**}}$	size N	$\frac{\text{count } N_r}{8}$	param	$\frac{1}{1}$ equally many routers on each die side?
5 5 5 5	2 3 4	66% 100% 133%	36 54 72	18 18 18 18	2 3 3 3	equally many router groups on each die side?
7 7 7	3 4 5	75% 100% 120%	150 200 250	50 50 50	5 5 5	There are few Slim Flies
11 11 11	4 5 6	66% 83% 100%	392 490 588	98 98 98	7 7 7	that satisfy various on-chip technological constraints
11 11	/ 8	116% 133%	686 784	98 98	7 7	

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Solutio Non-Prim	with				
Various	Slim Fly cor	nfigurations			
Network radix <i>k</i> '	Concen- tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right ceil **$	Network size N	Router count N <sub>r</sub>	Input param. q
6	2	66%	64	32	4
6	3	100%	96	32	4 🧰
6	4	133%	128	32	4e
12	4	66%	512	128	8
12	5	83%	640	128	8
12	6	100%	768	128	8
12	7	116%	896	128	8
12	8	133%	1024	128	8

71%

85%

100%

114%

810

972

1134

1296

162

162

162

162

9

9

9

9

...number of nodes/routers being a power of two?

> ...equally many cores on each die side?

...equally many routers on each die side?

...equally many router groups on each die side?



13

13

13

13

5

6

7

8

Solutio Non-Prim	with				
Various	Slim Fly cor	nfigurations			
Network radix <i>k</i> '	Concen- tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right ceil **$	Network size N	Router count N <sub>r</sub>	Input param. q
6	2	66%	64	32	4
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12	6	100%	768	128	8
12	7	116%	896	128	8
12	8	133%	1024	128	8

71%

85%

100%

114%

810

972

1134

1296

162

162

162

162

9

9

9

9

...number of nodes/routers being a power of two?

> ...equally many cores on each die side?

...equally many routers on each die side?

...equally many router groups on each die side?



SOLUTION	N: SLIM N 16 FINITE FI	oC Part	II ? confi	Are there	with	number of nodes/routers being a power of two?
Various S	Slim Fly con	figurations				equally many cores
Network radix k'	Concen- tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right ceil^{**}$	Network size N	Router count N <sub>r</sub>	Input param. q	equally many routers
6	2	66%	64	32	4	on each die side?
6	3	100%	96	32	4	
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12	8	133%	1024	128	8	
13	5	71%	810	162	9	
13	6	85%	972	162	9	
13	7	100%	1134	162	9	
13	8	114%	1296	162	9	



SOLUTION: SLIM NOC Part II NON-PRIME FINITE FIELDS Are there configurations with						number of nodes/routers being a power of two?
Various S	Slim Fly cor	nfigurations				equally many cores
Network radix k'	Concen- tration <i>p</i>	$p/\left\lceil \frac{k'}{2} \right ceil **$	Network size N	<b>Router</b> <b>count</b> N <sub>r</sub>	Input param. q	equally many routers
6	2	66%	64	32	4	on each die side?
6	3	100%	96	32	4	
6	4	133%	128	32	46	equally many router groups
12	4	66%	512	128	8	on each die side?
12	5	83%	640	128	8	
12	6	100%	768	128	8	
12	7	116%	896	128	8	
12	8	133%	1024	128	8	
2 How t	- o develon	71%	810	162	9	
	$r \cdot r$		9/2	162	9	
such a i	finite field		1134	162	У 0	
13	0	14%	1296	162	9	

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#### SOLUTION: SLIM NOC NON-PRIME FINITE FIELDS

How to develop such a finite field?



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## SOLUTION: SLIM NOC NON-PRIME FINITE FIELDS Recap: a finite field $\mathcal{F}_q$

Assuming *q* is **prime**:

 $\mathcal{F}_q = \mathbb{Z}/q\mathbb{Z}$ = {0,1, ..., q - 1} (with modular arithmetic).

**Example:** q = 5 50 routers

 $\mathcal{F}_5 = \{0, 1, 2, 3, 4\}$ 

How to develop such a finite field?



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Assuming *q* is **non-prime**:  $\mathcal{F}_q = \mathbb{Z}/q\mathbb{Z} = \{x_0, x_1, \dots, x_{q-1}\}$ 

Provide Statements

How to develop such a finite field?

...with instruction tables that define operations on the field.

**Example:** q = 9 162 routers  $\mathcal{F}_9 = \{0, 1, 2, u, v, w, x, y, z\}$ 

+ 0 1 2 u v w x y z	× 0 1 2 u v w x y z	elem -elem
0 012uvwxyz	0 0 0 0 0 0 0 0 0 0 0	0 0
1 1 2 0 v w u y z x	1 0 1 2 u v w x y z	1 2
2 2 0 1 w u v z x y	2   0 2 1 x z y u w v	2 0
u u v w x y z 0 1 2	u 0 u x 2 w z 1 v y	u x
vvwuyzx120	v	v z
wwuvzxy201	w  0 w y z 1 u v x 2	w y
x	x  0 x u 1 y v 2 z w	x u
y	y  0 y w v 2 x z u 1	y w
z	z  0 z v y u 2 w 1 x	z   v
Addition	Multiplication	Inverse



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Assuming $q$ is <b>prime</b> : $\mathcal{F}_q = \mathbb{Z}/q\mathbb{Z}$	<b>Example:</b> $q = 9$	162 routers				
= {0,1, (with mc arithmet	ck the paper for details 😳					
<b>Example:</b> $q = 5$ 50 routers $\mathcal{F}_{r} = \{0 \ 1 \ 2 \ 3 \ 4\}$	u u v w x y z 0 1 2 u 0 u x 2 w z 1 v v v w u y z x 1 2 0 v 0 v z w x 1 y 2 w w u v z x y 2 0 1 w 0 w y z 1 u v x x x y z 0 1 2 u v w x 0 x u 1 y v 2 z y y z x 1 2 0 v w u y 0 y w v 2 x z u z z x y 2 0 1 w u v z 0 z v y u 2 w 1	y u x 2 u v z 2 w y z w x u 1 y w 1 x z v				
J 5 – (U,I,Z,J,T)	Addition Multiplicatio	n Inverse				

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# How do we optimize the router microarchitecture for Slim NoC to provide high performance and high efficiency?

#### \*\*\*SPCL

SLIM NOC ROUTER MICROARCHITECTURE PERFORMANCE OPTIMIZATIONS Part III



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SLIM NOC ROUTER MICROARCHITECTURE PERFORMANCE OPTIMIZATIONS Part III

Let's leave the details for the paper and just focus on the core aspects ⓒ



and the states of





SLIM NOC ROUTER MICROARCHITECTURE PERFORMANCE OPTIMIZATIONS Part III



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#### SLIM NOC ROUTER MICROARCHITECTURE PERFORMANCE OPTIMIZATIONS Part III



Let's leave the details for the paper and just focus on the core aspects ⓒ

ENHANCEMENT 1: ELASTIC BUFFER LINKS [1] + ELASTISTORE [2]



[1] G. Michelogiannakis et al. Elastic-Buffer Flow Control for On-Chip Networks. HPCA'09.

[2] I. Seitanidis et al. ElastiStore: An Elastic Buffer Architecture for Network-on-Chip Routers. DATE'14.




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[4] S. Hassan and S. Yalamanchili. Centralized Buffer Router: A Low Latency, Low Power Router for High Radix NoCs. NOCS'13.





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### LET'S SUMMARIZE...



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### LET'S SUMMARIZE...











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## **EVALUATION**



## **EVALUATION METHODOLOGY: SENSITIVITY ANALYSES**



#### **METRICS:**

- LATENCY
- THROUGHPUT
- BUFFER AREA
- BUFFER SIZE
- STATIC/DYNAMIC POWER CONSUMPTION
- THROUGHPUT/POWER
- ENERGY-DELAY PRODUCT



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### TRAFFIC / WORKLOAD:

- UNIFORM RANDOM
- BIT SHUFFLE
- BIT REVERSAL
- ADVERSARIAL PATTERNS
- PARSEC/SPLASH TRACES



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#### **TOPOLOGY:**

CONCENTRATED MESH (CM)





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2D TORUS (T2D)



FLATTENED BUTTERFLY [1] (FBF)





[1] J. Kim, W. J. Dally, D. Abts. Flattened butterfly: a cost-efficient topology for high-radix networks. ISCA'07



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FLATTENED BUTTERFLY [1] (FBF)



FLATTENED
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   BUTTERFLY [1],
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- (BRIEFLY)
   HIERARCHICAL
   NOCs

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## **EVALUATION METHODOLOGY: SENSITIVITY ANALYSES**



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#### LAYOUT:

- GROUP
- SUBGROUP
- RANDOM
- NAIVE





#### LAYOUT:

- GROUP
- SUBGROUP
- RANDOM
- NAIVE







#### LAYOUT:

- GROUP
- SUBGROUP
- RANDOM
- NAIVE



### **ROUTER CYCLE TIME:**

- 0.4NS
- 0.5NS
- 0.6NS







#### LAYOUT:

- GROUP
- SUBGROUP
- RANDOM
- NAIVE



### **ROUTER CYCLE TIME:**

- 0.4NS
- 0.5NS
- 0.6NS



?

- 22NM
- 45NM



9



#### LAYOUT:

- GROUP
- SUBGROUP
- RANDOM
- NAIVE



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- 0.4NS
- 0.5NS
- 0.6NS



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- 22NM
- 45NM







- CENTRAL
- EDGE



#### LAYOUT:

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- SUBGROUP
- RANDOM
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TECHNOLOGY NODE:

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**TECHNOLOGY NODE:** 

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- 45NM







Edge

- SMART ON/OFF
- CENTRAL BUFFERS ON/OFF



#### LAYOUT:

- GROUP
- SUBGROUP
- RANDOM
- NAIVE



### **ROUTER CYCLE TIME:**

- 0.4NS
- 0.5NS
- 0.6NS



**INJECTION RATE:** ■ 0.01 - 0.95

### TECHNOLOGY NODE:

- 22NM
- 45NM





GLOBAL







- SMART ON/OFF
- CENTRAL BUFFERS ON/OFF



#### LAYOUT:

- GROUP
- SUBGROUP
- RANDOM
- NAIVE



#### **ROUTER CYCLE TIME:**

- 0.4NS
- 0.5NS
- 0.6NS

**INJECTION RATE: 0.01 – 0.95** 

TECHNOLOGY NODE:

- 22NM
- 45NM



GLOBAL

NETWORK SIZE (NODE COUNT):

- **200**
- **1**024
- 1296







- SMART ON/OFF
- CENTRAL BUFFERS ON/OFF



#### ROUTING

- MINIMUM STATIC
- **NON-MINIMUM ADAPTIVE**

#### LAYOUT:

- GROUP
- SUBGROUP
- Random
- NAIVE



### **ROUTER CYCLE TIME:**

- 0.4ns
- 0.5ns
- 0.6NS



**INJECTION RATE:** 0.01 – 0.95 

**TECHNOLOGY NODE:** 

- 22NM
- 45<sub>NM</sub>

### WIRE TYPE:



GLOBAL 



**NETWORK SIZE** (NODE COUNT):

- 200
- 1024
- 1296



3

8

9



- SMART ON/OFF
- CENTRAL BUFFERS ON/OFF



# **RESULTS: PERFORMANCE**

in-house simulator [1]

cm3: concentrated mesh, t2d3: torus,
pfbf3, pfbf4, fbf3: variants of Flattened Butterfly,
sn\_subgr: Slim NoC (the subgroup layout)

SMART LINKS: ON CENTRAL BUFFERS: ON NODE COUNT: 192/200



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## **RESULTS: AREA AND POWER CONSUMPTION** DSENT power simulator [1]

SMART LINKS: ON CENTRAL BUFFERS: ON NODE COUNT: 192/200, TECHNOLOGY NODE: 45NM



i-routers: routers (intermediate layer), a-routers: routers (active layer),
 RRg-wires: router-router wires (global layer), RNg-wires: router-node wires (global layer).
 [1] C. Sun et al. DSENT - A Tool Connecting Emerging Photonics with Electronics for Opto-Electronic Networks-on-Chip Modeling. NOCS'12.



## **RESULTS: AREA AND POWER CONSUMPTION** DSENT power simulator [1]

SMART LINKS: ON CENTRAL BUFFERS: ON NODE COUNT: 192/200, TECHNOLOGY NODE: 45NM

# Slim NoC is more efficient than high-radix designs



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## **RESULTS: THROUGHPUT / POWER (PARSEC/SPLASH)** in-house simulator [1]

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## **RESULTS: SCALABILITY**

Slim NoC is similarly advantageous when we move from 200 nodes to 1296 nodes (check the paper for details ⓒ)





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#### **O**THER RESULTS

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## EHzürich

# Slim NoC: A Low-Diameter On-Chip Network Topology for High Energy Efficiency and Scalability MACIEJ BESTA, SYEE

MACIEJ BESTA, SYED MINHAJ HASSAN, SUDHAKAR YALAMANCHILI, RACHATA AUSAVARUNGNIRUN, ONUR MUTLU, TORSTEN HOEFLER

Georgia







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## **ANALYSIS: DIAMETER-2 SLIM FLY**



#### **ANALYSIS: DIAMETER-2 SLIM FLY**

Lowest latency
Better throughput than Dragonfly
Almost-the-best throughput

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#### **ANALYSIS: DIAMETER-2 SLIM FLY**

~25-30% cost reduction vs. second-best topology (Dragonfly)

Lowest latency
Better throughput than Dragonfly
Almost-the-best throughput



## ANALYSIS: DIAMETER-2 SLIM FLY COST OF NETWORK CONSTRUCTION





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## ANALYSIS: DIAMETER-2 SLIM FLY COST OF NETWORK CONSTRUCTION



Number of endpoints [thousands]





ANALYSIS: DIAMETER-2 SLIM FLY PERFORMANCE (UNIFORM RANDOM)















































ANALYSIS: DIAMETER-2 SLIM FLY PERFORMANCE (UNIFORM RANDOM)





Lowest latency
Better throughput than Dragonfly
Almost-the-best throughput





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Slim Fly:

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#### **Dragonfly:**







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"q": the input parameter that determines the network structure. Formally, the base of a finite field (Slim Fly uses <u>prime</u> q; the corresponding field: {0, 1, ..., q-1}.



**Dragonfly:** 




## SLIM FLY ON CHIP – FIRST ATTEMPT STRUCTURE INTUITION

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## **Dragonfly:**



two groups



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## SOLUTION: SLIM NOC

NEW COST AND AREA MODELS, NEW LAYOUTS

Let us see some layouts

What difference do they make for lengths of wires?



Distance ranges

**Figure 6.** (§ 3.3) Distribution of link distances in SNs. A bar associated with a distance range X illustrates the probability that, for a given layout, two routers are connected with a link that has the distance falling within X. Bars of different colors are placed pairwise so that it is easier to compare the subgroup and group layouts.



## SOLUTION: SLIM NOC

NEW COST AND AREA MODELS, NEW LAYOUTS

Let us see some layouts



Distance ranges

What difference do they make for lengths of wires?

The "group layout" (sn\_gr) is best for 1296 nodes The "subgroup layout" (sn\_subgr) is best for 200 nodes



## SOLUTION: SLIM NOC NON-PRIME FINITE FIELDS

How to develop such a finite field?

The second second



## SOLUTION: SLIM NOC NON-PRIME FINITE FIELDS

Recap: a finite field  $\mathcal{F}_q$ Assuming q is prime:  $\mathcal{F}_q = \mathbb{Z}/q\mathbb{Z}$  $= \{0, 1, ..., q - 1\}$ (with modular

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How to develop such a finite field?

A CONTRACTOR



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**Example:** q = 5 50 routers

 $\mathcal{F}_5 = \{0, 1, 2, 3, 4\}$ 

How to develop such a finite field?

Participation (State)



SOLUTION: SLIM NOC NON-PRIME FINITE FIELDS

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Assuming *q* is **non-prime**:  $\mathcal{F}_q = \mathbb{Z}/q\mathbb{Z} = \{x_0, x_1, \dots, x_{q-1}\}$  How to develop such a finite field?

...with instruction tables that define operations on the field.



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**Example:** q = 9 162 routers  $\mathcal{F}_9 = \{0, 1, 2, u, v, w, x, y, z\}$ 



**SOLUTION: SLIM NOC NON-PRIME FINITE FIELDS** *Recap: a finite field*  $\mathcal{F}_a$ Assuming *q* is **prime**:  $\mathcal{F}_q = \mathbb{Z}/q\mathbb{Z}$  $= \{0, 1, \dots, q-1\}$ (with modular arithmetic).

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 $\frac{+|0|1|2|u|v|w|x|y|z}{0|0|1|2|u|v|w|x|y|z|x}$ 
 $\frac{0|0|1|2|u|v|w|x|y|z}{1|1|2|0|v|w|u|y|z|x}$  

 2|0|1|w|u|v|z|x||2|0| 

Addition

w w u v z x y 2 0 1 x x y z 0 1 2 u v w y y z x 1 2 0 v w u z z x y 2 0 1 w u v



SOLUTION: SLIM NOC NON-PRIME FINITE FIELDS Recap: a finite field  $\mathcal{F}_q$ Assuming q is prime:

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+ 0 1 2 u v w x y z	× 0 1 2 u v w x y z
0 012uvwxyz	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 2 0 v w u y z x	1   0 1 2 u v w x y z
2 2 0 1 w u v z x y	2   0 2 1 x z y u w v
uuvwxyz012	u  0 u x 2 w z 1 v y
vvwuyzx120	v
wwwvzxy201	w  0 w y z 1 u v x 2
x x y z 0 1 2 u v w	x  0 x u 1 y v 2 z w
y y z x 1 2 0 v w u	y  0 y w v 2 x z u 1
z   z x y 2 0 1 w u v	z  0 z v y u 2 w 1 x

Addition Multiplication



SOLUTION: SLIM NOC NON-PRIME FINITE FIELDS Recap: a finite field  $\mathcal{F}_a$ 

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+ 0 1 2 u v w x y z	× 0 1 2 u v w x y z	elem -elem
0 012uvwxyz	0 0 0 0 0 0 0 0 0 0 0	0 0
1 1 2 0 v w u y z x	1 0 1 2 u v w x y z	1 2
2 2 0 1 w u v z x y	2   0 2 1 x z y u w v	2 0
u u v w x y z 0 1 2	u	u x
vvwuyzx120	v	v z
wwuvzxy201	w  0 w y z 1 u v x 2	w y
x	x  0 x u 1 y v 2 z w	x u
y	y  0 y w v 2 x z u 1	y w
z	z  0 z v y u 2 w 1 x	z   v
Addition	Multiplication	Inverse



## SOLUTION: SLIM NOC NON-PRIME FINITE FIELDS

Recap: a finite field  $\mathcal{F}_q$ 

Assuming a is prime: Generate with an exhaustive search, or use a construction (based on polynomials arithmetic).

**Example:** q = 5 50 routers

 $\mathcal{F}_5 = \{0, 1, 2, 3, 4\}$ 

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	0 012uvwxyz	0 0 0 0 0 0 0 0 0 0 0	0 0
-	1 1 2 0 v w u y z x	1 0 1 2 u v w x y z	1 2
	2 2 0 1 w u v z x y	2 0 2 1 x z y u w v	2 0
	u u v w x y z 0 1 2	u	u x
	vvwuyzx120	v  0 v z w x 1 y 2 u	v z
	wwuvzxy201	w  0 w y z 1 u v x 2	w y
	x	x  0 x u 1 y v 2 z w	x u
	y y z x 1 2 0 v w u	y  0 y w v 2 x z u 1	y w
	z	z  0 z v y u 2 w 1 x	z V
	Addition	Multiplication	Inverse



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## **EVALUATION METHODOLOGY** SIMULATION INFRASTRUCTURE





## **EVALUATION METHODOLOGY** SIMULATION INFRASTRUCTURE

Performance

Cycle-accurate simulations (in-house simulator [1], Booksim [2])



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PERFORMANCE

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## **RESULTS: AREA AND POWER CONSUMPTION**

SMART LINKS: ON CENTRAL BUFFERS: ON NODE COUNT: 192/200





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Slim NoC ensures the lowest latency



#### SMART LINKS: ON CENTRAL BUFFERS: ON

#### **EVALUATION**

**SELECTED OTHER INSIGHTS** 



Node count: 54



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NODE COUNT: 192/200







1 Intra-group connections





- 1 Intra-group connections
- Path of length 1 or 2between two routers





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Inter-group connections (different types of groups)





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**1** Select a prime power q





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E Example: q = 550 routers network radix: 7  $\mathcal{F}_5 = \{0,1,2,3,4\}$ 






















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$$X = \{1, \xi^2, ..., \xi^{q-3}\}$$
  
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Take Routers  $(0,0,.)$   
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Self Contractions



7 Inter-group connections Router  $(0, x, y) \leftrightarrow (1, m, c)$ 



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7 Inter-group connections Router  $(0, x, y) \leftrightarrow (1, m, c)$ iff y = mx + c



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E Example: 
$$q = 5$$
  
Take Router (1,0,0)  $m = 0, c = 0$ 

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Carlo and and a







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#### **DIAMETER-2 SLIM FLY**



E Example: 
$$q = 5$$
  
Take Router (1,0,0)  
 $(1,0,0) \leftrightarrow (0, x, 0)$   
Take Router (1,1,0)  $m = 1, c = 0$   
 $(1,0,0) \leftrightarrow (0, x, x)$ 

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ATTACHING ENDPOINTS: DIAMETER 2





**ATTACHING ENDPOINTS: DIAMETER 2** 

1 Get load *I* per router-router channel (average number of routes per channel)





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 $l = \frac{total \ number \ of \ routes}{total \ number \ of \ channels}$ 





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Make the network balanced, i.e.,:





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concentration = 33% of router radix



AVERAGE DISTANCE



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**AVERAGE DISTANCE** 

Uniform random traffic using minimum path routing

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**AVERAGE DISTANCE** 

Uniform random traffic using minimum path routing



New York Concerns



**BISECTION BANDWIDTH (BB)** 



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**BISECTION BANDWIDTH (BB)** 

\*BB approximated with the Metis partitioner [1]



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